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A Guide to the math component of the Digital SAT

The Objective of this Book

In any high stakes test (especially the digital SAT) There are three critical components that the student needs to master:

- Subject Knowledge
- Time Management
- Carelessness

We have created an online practice application: https://www.satdiagnostics.com that addresses the three components and is closely linked to this book. It is private and anonymous. There is no user id, password or tracking.

The primary goal is to provide a plan of attack. If you have seen and practiced enough problems, then in the first few seconds you can formulate:

- The concept behind the problem
- The first few steps leading to a solution
- Misdirection and pitfalls (words or logic)

We start by examining the possible algorithms that describe how the College Board generates module I and module II-easy or module II-hard questions (weights and distribution) and the effect on a student's overall math score

This book is the culmination of years of working with some of the brightest and most motivated students in America.

Thank you in advance for using this book.

Frank Mitchell Rubin Los Angeles, CA Jan 2025

Resources

The list below covers our recommended resources:

I. https://satsuitequestionbank.collegeboard.org/ - This is a downloadable set of paper-based questions from previous SAT tests that are selectable by topic and difficulty (there are 343 difficult questions).

II. https://bluebook.collegeboard.org/students - This is the entre into College Boards six simulated online tests.

III. https://www.khanacademy.org/ - Khan academy is a free site with close ties to the College Board. Also investigate their trial 'boot camp'.

IV. https://testinnovators.com/ They charge approximately $200 for a complete package (verbal and math). The math problems closely emulate the College Board blue book and there are 10 simulated tests. The explanations are concise and very clear. In addition, they have many supporting videos.

Our recommendation is to first complete items IV and some of the more difficult problems from I

Then work on II and III. The results of each Blue Book test direct you to appropriate problems in Khan.

The scoring algorithm behind the Digital SAT

The 2024 Online Digital SAT incorporates a two-stage hybrid adaptive test format. This means that the score on the first module (mod1) of the Math component dictate whether the second mod the student gets is easy or hard. Each mod contains 22 questions (only 20 are scored – the other two are experimental to help design future tests). ***The questions themselves are NOT adaptive BUT they are WEIGHTED.***

This version dramatically affects students who are either: (1) not prepared for the most difficult questions or (2) tend to be careless (due more to misreading then calculation). If the student makes it to mod2-hard, they may likely score between 700 – 800. Alternatively, mod2-easy may limit them to the 500 – 600 range. This is all due to the weighting algorithms.

In essence, if you don't move to mod2-hard your grade could result in a 200-point difference. This is a direct result of the combination of question weights and frequency of hard questions in mod2 hard versus easy.

A Plausible Explanation

College board not only does not publish their algorithms or weights (of questions). They may also change the weights and their distribution as data from previous test results become available. Therefore, we will present a hypothetical model which helps explain the importance of understanding the theory behind the algorithm.

Assumptions:

- Question Weights:
  - Difficult = 1.2

- Medium = 1.0
- Easy = 0.8

- Distribution of weighted questions mod1, mod2H and mod2E
  - Mod1: H = 7, M = 7, E = 6
  - Mod2.H: H = 12, M = 5, E = 3
  - Mod2.E: H = 3, M = 10, E = 7
- Percent Correct
  - Student A: H = 0.8, M = 0.9, E = 1.0
  - Student B: H = 0.6, M = 0.8, E = 0.9

Results: As reported in the simulation below. With both weighting and non-weighting, the scores are similar, but student A's total score would be in the high 700's while student B would be in the low to mid 500's. A two-hundred-point difference.

Therefore, the **primary objective** of this book and the associated software applications is to increase the odds of a student moving from: **Module 1 to Module 2 Hard**

user inputs	factors	weights		mod 1 dist	mod 2 Hard	mod 2 easy			
Difficult	0.2	1.2		7	12	3		NON WEIGHTED	
Medium	1	1		7	5	10	raw score	H - range	E - range
Easy	0.2	0.8		6	3	7	26	580 - 640	430 - 490
factors are relative to medium							27	600 - 660	450 - 510
				% correct (as decimal)			28	610 - 670	460 - 520
			mod 1	mod 2	mod 2		29	630 - 690	470 - 530
	Group		difficult	medium	easy		30	640 - 700	490 - 550
	m2-H students		0.8	0.9	1		31	660 - 720	500 - 560
	m2-E students		0.6	0.8	0.9		32	680 - 740	520 - 580
							33	700 - 760	530 - 590
weighted			mod 1	mod 2	raw score	raw score	34	720 - 780	580 - 640
			raw scor	mod II Hard	Mod II Easy	tot mod I & II	35	740-800	540 - 620
	m2-H students		17.82	18.42	X	36.138	36	760 - 800	580 - 660
	m2-E students		14.96	X	15.2	30.1192	37	770 - 800	x
							38	780 - 800	x
non weighted			mod 1	mod 2	raw score	raw score	39	780 - 800	x
			raw scor	mod II Hard	Mod II Easy	tot mod I & II	40	780 - 800	x
	m2-H students		19.4	17.1	X	36.891			
	m2-E students		15.2	X	16.1	31.147			

It is critical that the student master the hard problems.

The Trichotomy Principal

There are three different cases to consider:

- Quadratic Equations with a variable constant or coefficient.
- System of Linear Equations.
- Comparison of two Linear Expressions.

And are three outcomes with this principal:

- Two or more real solutions
- Exactly one real solution
- No real solutions, but in the case of quadratics, there may be complex (imaginary) solutions.

Case 1. Quadratic equation with different outcomes

Concept: Given a quadratic equation in standard form:

$$y = ax^2 + bx + c$$

This represents geometrically a parabola, which faces up if $a > 0$ (vertex = min value) or down if $a < 0$ (vertex = max value).

An alternative to factoring (if possible) is using the quadric solution:

$$x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$$

Let D (the discriminant) = $b^2 - 4ac$.

- If $D > 0$ then the quadratic will intercept the x-axis at two different points giving two solutions $x = (-b \pm \sqrt{D})/2a$.

- If D = 0 then then the quadratic will intercept the x-axis at a single point giving a single solutions x = –b/2a which is referred to as the line of symmetry (the parabola is symmetric with respect to this line).
- If D < 0 then, since the square root cannot contain a negative number, there are no Real solutions. It's a perfectly good parabola, it just lies either totally above or below the x-axis and has complex (imaginary) solutions.

Example 1. Given $f(x) = 2x^2 + 4x + d$. For what value of d, will f(x) have two distinct real solutions?

Concept: A quadratic equation has **two** real solutions when the discriminant > 0.

Solution: Discriminant = $b^2 - 4ac > 0$ Therefore $16 – 8d > 0$, and $d < 2$. Practice

Example 2. Given $f(x) = 2x^2 + 4x + d$. For what value of d, will f(x) have exactly one solution?

Concept: A quadratic equation has **one** real solution when the discriminant = 0.

Solution: The discriminant = $b^2 - 4ac = 0$ Therefore $16 – 8d = 0$, and $d = 2$. Practice

Example 3. Given $f(x) = 2x^2 + 4x + d$. For what value of d, will f(x) have no real solutions (just complex)?

Concept: A quadratic has no real solutions when the discriminant < 0.

Solution: The discriminant = $b^2 - 4ac < 0$ Therefore $16 – 8d < 0$, and $d > 2$. **Practice**

Case II. System of Linear Equations with Different Outcomes

The Concept: Given a System of linear equations in standard form:

$$\boldsymbol{ax + by = c}$$

$$\mathbf{dx + ey = f}$$

This represents, geometrically, a two lines in the x-y plane. There are three outcomes:

- The lines are parallel which implies that the slopes are equal, and the y-intercepts are different. Therefore, there are no solutions – the lines don't meet.
- The lines are collinear. The equations are identical by some multiple. Therefore, they have an infinite number of solutions
- The lines intersect. The slopes are different, and they have exactly one solution

For a linear equation in standard form, the slope equals -a/b and the y-intercept (c/b) is obtained by letting $x = 0$ and the x-intercept is obtained by letting $y = 0$.

Example 4. Given $ax + 4y = 102$ and $8x + 16y = 30$. For what value of a, will the system have no solution?

Solution: To have no solution the lines must be parallel. This implies that the slopes are equal, and the y-intercepts are different. Comparing the slopes: **-a/4 = -8/16 => a = 2.** Comparing the y-intercepts: **12/4 ≠ 40/16** Practice

Example 5. Given ax + 4y = 10 and 8x + 16y = 40. For what value of a, will the system have an infinite number of solutions?

Solution: To have an infinite number of solutions the lines must be the colinear. This implies that both the slopes and y-intercepts are the same. Comparing the slopes: **-a/4 = -8/16 => a = 2.** Comparing the y-intercepts: **10/4 = 40/16**. Another way to solve this is to show that one equation is a multiple of the other. Practice

Example 6. Given ax + 4y = 12 and 8x + 16y = 40. For what value of a, will the system have exactly one solution?

Solution: To have an exactly one solution the lines must intersect. This implies that the slopes are different with no restrictions on the y-intercept. Comparing the slopes: **-a/4 = -8/16 => a ≠ 2**. Practice

Case III. Comparison of two Linear Expressions with different outcomes

The Concept: Given two linear expressions: ax + b = cx + d. There are three possible outcomes:

- a = c and b = d. The expressions are the same and the equality will hold regardless of the value of x. Therefore, there are an **infinite number of solutions.**

- a = c and b ≠ c. The constants are different. Since b ≠ c there are **no values of x resulting in equal expressions**.
- a ≠ c. The coefficients of x are different and **there is a unique** solution: x = (d – b)/(a – c).

Example 7. Given ax + 5 = 3x + 5. For what value of a, will the system have infinitely many solutions.

Solution: if a = 5 then the expressions are the same.

Example 8. Given ax + 7 = 3x + 5. For what value of a, will the system have no solutions.

Solution: if a = 3 then the expressions reduce to 7 ≠ 5, which is not true. **Practice**

Example 9. Given ax + 2 = 3x + 5. For what value of a, will the system have exactly one solution.

Solution: if a ≠ 3 then the expressions have exactly one solution .

Linear Equations and Systems

The Concept: There are two primary ways in which a linear equation can be expressed:

- Standard Form: $ax + by = c$
- Slope Intercept: $y = mx + b$, where m is the slope and b the y-intercept.

In addition, Unit measures: miles/gal or gal/miles are equivalent to slopes. Miles/gallon measures how many miles we can get on just one gallon of gasoline, while gallons/mile measure how many gallons we need to travel just one mile.

An Example of standard form:

- There are x small bottles each containing 'a' fluid ounces and y large bottle each containing 'b' fluid ounces the combined number of fluid ounces of both bottles is 'c'. This is reflected in the equation: **$ax + by = c$.**

An Example of Slope Intercept:

- A plumber charges \$c comprised of a flat fee of \$b (just for coming out) and hourly fee of \$m. This is reflected in the equation: c = m*h + b, where h is the number of hours spent at the house.

Example 10: A grocery carries two types of fruits Apples (A) at $5 per pound and Peaches (P) at $3 per pound. In total 50 pounds of fruit were sold

- What is the maximum and minimum totals sales?
- If 10 pounds more apples were sold then peaches, what would be the total received?

Solution: For the first part the equation (in standard form):

T = 5A + 3P

- The **maximum** total occurs when we sell 50 pounds of just apples = $250. The **minimum** total occurs when we sell 50 pounds or just peaches = $150
- Ten pounds more…, translates to A = P + 10. But remember, A + P = 50. Therefore P + 10 + P = 50. Hence, 2P = 40 and P = 20 and A = 30. Therefore, the total = $5*30 + $3*20 = $150 + $60 = $210

Example 11: Given a linear equation f(x) such that f(2) = 5 and f(7) = 20. If another linear equation g(x) is perpendicular to f(x) and g(3) = 12, what is g(12)?

Solution: The approach is to first find the slope of f(x) and then find the slope of g(x) which is the negative reciprocal (a property of perpendicularity). Finally, plug the given point on g(x) to get its linear equation.

- The slope of f(x) = [f(7) – f(2)]/[7 – 2] = [20 – 5]/5 = 15/5 = 3. The slope of g(x) = –1/3.
- Therefore g(x) = –(1/3)x + b, where b is the y-intercept.
- To determine b, since g(3) = 12, plug in the point (3,12) into g(x) = –(1/3)x + b
- 12 = –(1/3)(3) + b, which implies that b = 13
- Therefore, g(x) = –(1/3)x + 13
- And g(12) = –(1/3)12 + 13 = 9. Practice

Example 12: A taxicab travels **k miles per week**. It gets **j miles per gallon** of gas. Gas cost **$n per gallon**. If the driver wishes to save $p dollars, what formula could be used to find how many less miles they should drive.

Solution: To turn a problem from theoretical (with only variables) to a more intuitive one, plug in numbers for the variables.

- First determine how many gallons (or fractional part) is required to travel 1 mile = 1/j (if you get 5 mpg then you can travel 1 mile on 1/5 of a gallon).
- Since gas cost $n per gallon, then the cost to travel one mile = $n(1/j) (suppose n = $10, then the cost = $10(1/5) = $2 to travel 1 mile).
- Finally, to solve for how many less miles to save $p, let x = the number of less miles they need to drive. Therefore: x = p/(n/j) = pj/n is the number of less miles required to save $p. Suppose that p = $40. According to our scenario it cost 10/5 ($2) to travel 1 mile then: x = $40/(10/5) = ($40/2) = 20 less miles.

Example 13: The cost to enter an amusement park is composed of a onetime fee of $70 plus a per person fee of $20 for the first 15 people in the group and an additional $10 for each additional person. if n > 15 people are in the group, what is linear equation c(n) that give the total cost for admission for this group?

Solution: Start with a literal translation:

- C(n) = 70 + 20*15 + 10(n – 15)
- C(n) = 370 + 10n – 150
- C(n) = 10n + 220

Example 14: 3x + 6y = 12 is translated down 5 units. What are the coordinates of the x and y intercepts of the new equation

Solution: A vertical translation produces a parallel line with the same slope and the intercept 5 units less. The slope of both linear equations equals $-3/6 = -1/2$. The old intercept occurs when $x = 0$ => y-intercept = 2. Therefore, the new y-intercept = 2 - 5 = -3. Therefore, the new equation is:

$$Y = (-1/2)x - 3$$

The y-intercept = -3. The x-intercept occurs when y=0 and equals -6.

Example 15: Given a triangle with two sides: 8 and 13, what is the minimum and maximum that the third could be?

The Concept: The Triangle inequality states that given three sides of triangle: a, b and c, any side, for example c, must be strictly less than the sum of the other two sides and strictly greater than their difference: $|b - c| < a < b + c$. Note that we use an absolute value on the left side, since with variables we don't know which is greater but the difference must be positive.

Solution: let the third side be denoted by x, then:

$$13 - 8 < x < 13 + 8 \Rightarrow 5 < x < 21$$ Practice

Example 16: **Tom gets $7 per hour for the first 10 hours and $10 per hour after that in each week. If tom saves 70 % or his wages, what is the minimum number of hours he must work to save $200.**

Solution: Start with a literal translation:

$$0.7*[7h + 3(h - 10)] = 200$$

$$0.7(10h - 30) = 200$$

$$7h - 21 = 200$$

$$n = 221/7$$

The minimum number of hours = 32 (round up)

Quadratics Equations

Concepts: quadratic equations can take on many forms.

1) $f(x) = ax^2 + bx + c$ ... standard form
2) $f(x) = a(x - h)^2 + k$ … vertex form, where (h, k) is the vertex
3) $f(x) = a(x - s1)(x - s2)$ ... factored form, where s1 and s2 are the x-intercepts if there are any.

In all three cases, if $a > 0$ the parabola faces up – a minimum point, otherwise down – a maximum. One of the key coordinates of a quadratic equation is its vertex, which is either a min or max point. The x-coordinate of the vertex can be derived from the line of symmetry. For the equation in (1) above: $x = -b/2a$. Alternatively for (3): the line of symmetry equals the average of the x-intercepts. The y-coordinate can be evaluated by replacing x in f(x) with the x-coordinate of the vertex.

Example 17: **Given the quadratic** $y = x^2 + 2x - 15$. What is its minimum or maximum value and where does it occur?

The Concept: Consider the solution to a quadratic in standard form: $y = Ax^2 + Bx + C$. (Note that geometrically, a quadratic is a parabola.)

$$y = [-B \pm \sqrt{(B^2 - 4AC)}]/2A$$

Then the line of symmetry: $x = -B/2A$. The x value is also the **x value of the vertex.** If the coefficient A is positive then the parabola faces upward and has a minimum, otherwise the reverse. This x coordinate specifies **WHERE** the minimum occurs. To find **what** the value is (the y-coordinate of the vertex), plug the x-value into the quadratic equation.

Solution: In this example, $x = -2/2 = -1$. Plugging this back into the equation:

$$\mathbf{y = (-1)^2 + 2(-1) - 15 = -12}$$

Therefore, the vertex = (-1 , -12)

Because A > 0 the minimum value is -12 and occurs at x = -1

Practice

Example 18: Given the quadratic $y = (x - 3)(x + 5)$. Where does the minimum occur what is its value?

Solution: This is the (x , y) vertex (where , what). x equals the average of the x-intercepts = $[3 + (-5)]/2 = -1$ (where).

$Y = (-1 - 3)(-1 + 5) = (-4)(4) = -16$ (what).

The vertex is (–1,–16) Practice

Example 19: $f(x) = (x - 3)(x + 5)$ and $h(x) = f(x - 2)$, what is the value of h(0) ?

Solution: Like example 18, the vertex is (–1, –16). Therefore, the equation in vertex form is $\mathbf{f(x) = (x+1)^2 - 16}$. And f(x–2) results in a horizontal shift of 2 units to right with a vertex of (1,16). Therefore, $\mathbf{h(x) = (x-1)^2 - 16}$.

Therefore h(0) = 1 – 16 = –15

Example 20: Given the formula for the height of a projectile: $h(t) = 16t - 32t^2 + 20$. What is its maximum height. At what time does it reach It and when does it hit the ground?

Solution: Re writing the equation in standard form:

$$H(t) = -32t^2 + 16t + 20$$

The x value of the vertex = $-B/2A = -16/(-64) = 1/4$. Plugging in: $H(1/4) = -32(1/16) + 16(1/4) + 20 = 22$. Therefore, the vertex = (1/4 , 22) indicating that its maximum height is 22 units and it occurs in ¼ second.

It hits the ground when y = 0. The solution can be determined by soling the quadratic equation (which we leave to you) **$[-B \pm\sqrt{(B^2 - 4AC)}]/2A$**

Example 21: Given the quadratic equation:

$$\mathbf{f(x) = x^2 - 6x + 15}$$

If a = sum of the x-intercepts and b = the product, what is the value of a+b and a*b.

Concept: If r1 and r2 are the x-intercepts (the solutions) of the quadratic equation $f(x) = x^2 + bx + c$. Then:

$$f(x) = (x - r1)(x - r2) \Rightarrow f(x) = x^2 - (r1 + r2)x + r1r2.$$

$$\mathbf{b = -(r1 + r2) \text{ and } c = r1r2}$$

Solution: a = –(–6) = 6 and b = 15. Therefore, a + b = 21 and a*b = 15.

Example 22: If $f(x) = 3(x-2)^2 + 5$ is the vertex form of the quadratic equation. And $f(x) = ax^2 + bx + c$ is its standard form, what are the values of a, b and c?

Solution: Expand the vertex form into the standard form and compare the coefficients and constant.

$$f(x) = 3(x^2 - 4x + 4) + 5 \Rightarrow f(x) = 3x^2 - 12x + 17$$

Comparing, a = 3, b = −12 and c = 17

Example 23: Given the exponential equation:

f(t) = a(0.35)∧t (note ^t means raised to power t)

where t represents the time-period in months. If f(0) = 300, what is f(2).

Concept: In the exponential equation f(t) = a(1 ± b)^t:

- a represents the initial value when t = 0.
- b represents the rate. 1 ± b represents 100% of the prior iteration ± an additional b%.
- If b>0 then the this is a growth function. Otherwise, decay.

Solution: Since f(0) = a(0.35)^0 = a (anything to the 0^{th} power = 1) then a = 300. Therefore, f(2) $300(.35)^2$ = 300(.1225) = 36.75.

Example 24: Given the exponential equation:

f(t) = a(b)^t. If f(0) = 300 and f(1) = 600, then what does f(2) equal?

Solution: This is one step more complicated then example 23. After determining that a = 300, we have f(t) = 300*b^t.

Since f(1) = 600 we have 600 = 300*b. Therefore, b = 2. Finally, f(2) = $300*2^2$ = 1200.

Example 25: F(t) = 1000(0.25)^(t/6) describes the depreciated value of a car after t months. If the value decreases each year by p% of its value from the previous year, what is the value of p?

Solution: t/6 implies that every 6 months its value is ¼ of what it was 6 months ago. Therefore, each year the value of the car is ¼ * ¼ = 1/16 = 6.25% of the previous years value.

Example 26: f(x) = 8(3^x). g(x) = f(x+3). What does g(2) equal?

Solution: g(2) = f(2+3) = f(5) = 8(3^5) = 8 * 243 = 1,944

Example 27: **What is the solution(s) to the equation:**

$$\sqrt{(x^2)} = 4x - 3$$

Concept: $\sqrt{(x^2)} \neq x$ because if $x < 0$ then the left side is positive while the right side is negative. Instead

$\sqrt{(x^2)} = |x|$

Solution: $\sqrt{(x^2)} = 4x - 3 \Rightarrow |x| = 4x - 3$. Therefore, either

$x = 4x - 3$ or $x = 3 - 4x$. So $x = 1$ or $3/5$

Example 28: Given the equation:

$$(\sqrt{x^3})/(\sqrt[3]{x^2}) = x \wedge (a/b).$$ What is a/b?

Solution: Change the left side of the equation to $x^{(3/2)} / x^{(2/3)} = x^{(3/2 - 2/3)} = x^{(5/6)}$. Therefore $a/b = 5/6$

Example 29: Simplify, $\sqrt[3]{(64k^3)}(\sqrt{(64k)})^2$

Solution: Simplifying: $(4k)(64k) = 256k^2$

Non-Linear Equations

Example 30: **If** $f(x) = a/(x + b)$ and $g(x) = f(x+5)$, and $h(x) = f(x) + 5$. What are the vertical asymptotes of: $f(x)$, $g(x)$ and $h(x)$?

Solution: Vertical Asymptotes (va) occur when the denominator of a rational expression equal zero. In this example, this occurs when $x = -b$. $g(x)$ represents a horizontal shift of 5 units to the left. Therefore, $x = -b - 5$ represents its va. Finally, $h(x)$ represents a vertical shift of 5 units upward and hence has no effect.

Example 31: Solve: $(8x^2)/(x^2-16) - (4x)/(x + 4) = 2/(x - 4)$

Solution: note that the denominators represent the difference of squares: $x^2-16 = (x+4)(x-4)$. Therefore, if we multiple the entire equation by x^2-16 (which removes all denominators), we have:

$$8x^2 - 4x(x - 4) = 2(x + 4)$$

Simplifying, $4x^2+14x - 8 = 0 \Rightarrow 2x^2+7x - 4 = 0$

$(2x - 1)(x + 4) = 0$, $\mathbf{x = \frac{1}{2}}$ **or** $\mathbf{-4}$

Example 32: The following equation is true for all values of x: $(ax + b)(4x^2 - 6x + 3) = 16x^3 + 12x^2 - 7x + 21$. What is the value of $a + b$?

Solution: Expand the left side and compare the coefficient of x^3 to 15 and the constant to 20, resulting in $4a = 16$ and $3b = 21$. Therefore, $a = 4$, $b = 7$ and $a + b = 11$.

Example 33: Given the equation:

$3/(x - 4) + 2/(x - 2) = (ax + b)/(x - 4)(x - 2)$

Is true for all values of $x > 4$, what is the value of ab?

Solution: Multiplying the entire equation by $(x - 4)(x - 2)$, the resulting equation has no denominator:

$$3(x - 2) + 2(x - 4) = ax + b$$

$$5x - 14 = ax + b$$

Therefore $a = 5$ and $b = -14$ and $ab = -70$

Ratios, Proportions, Probability

Example 34: the ratio of a to b is 3:7 and the ratio of b to c is 14:20. What is the ratio of a to c?

Solution: Change to ratio of a:b from 3:7 to 6:14. Now the middle number are the same and we can form the ratio: a:b:c = 6:14:14:20. Therefore a:c = 6:20 = 3:10

Example 35: A is 60% less then B which in turn is 40% greater than C. What is the percent of A to C?

Solution: Translate this literal $A = (1-0.6)B = (0.4)B = (1+0.4)(0.4)C = (1.4)(0.4)C = 0.56C = 56\%$ of C

Example 36. The population of a town is 15,000. A random sample of 500 is taken. From this sample, the probability that a person is less than 30 years old is 20% with a margin of error of 0.5%. What is the least number of people in the general population that could be over 30 years old?

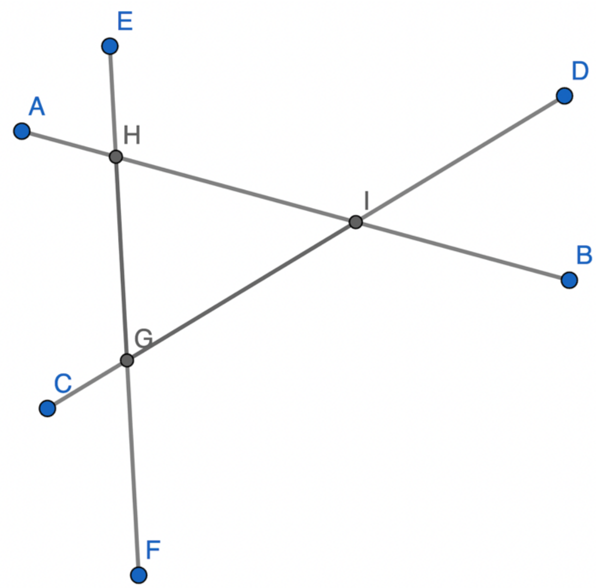
Solution: Start with the sample. If the probability of being less the 30 is 20%, then the probability of being over 30 is 80% with the same margin of error. Therefore, 0.8 * 15,000 = 12,000 and 0.5% of 15,000 = 0.005 *15,000 = 75.

Finally, the least number of people that could be over 30 = 12,000 – 75 = 11925.

Example 37. Group A has 8 people with an average weight of 140 pounds. Group B has 6 people with an average of 110 pounds. If Groups A and B are combined, what is the average weight?

Solution: Create a new average from scratch (don't average averages). New average = (8*140 + 6*110)/(8+6) = (1,120 + 660)/14 = 1780/14 ≈ 127.14

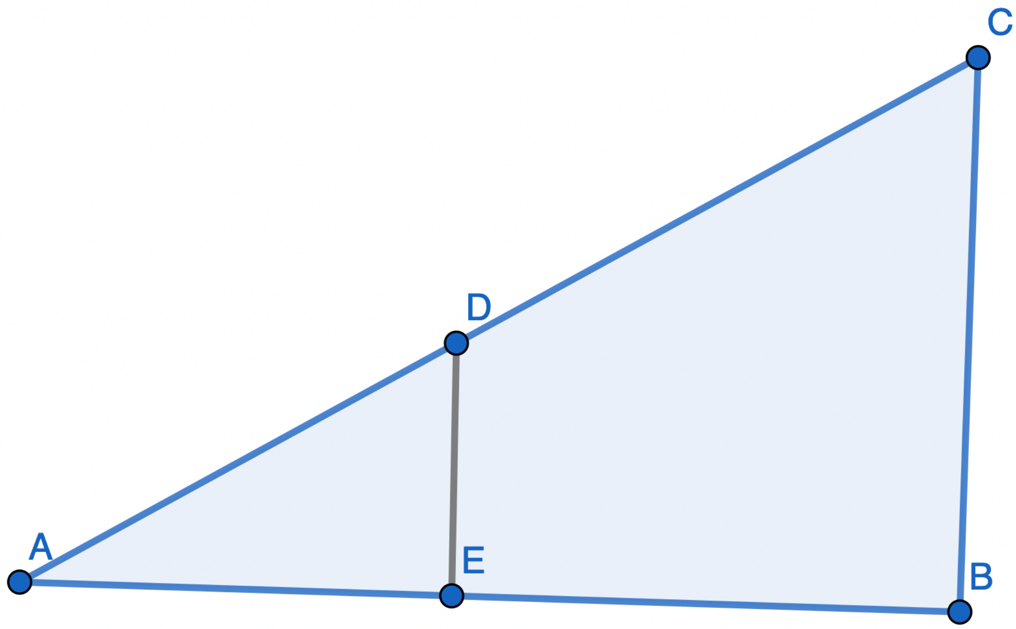
Example 38.



Given the above diagram illustrating three intersecting lines. What is the sum of <FGI, <DIH and <EHJ ?

Solution: If we try adding the supplements of the interior angles of the triangle, we may mistakenly conclude that since we don't know the interior angles, this problem can't be solved. **But there is a theorem in geometry that states that sum of the exterior angles any polygon equal 360°.**

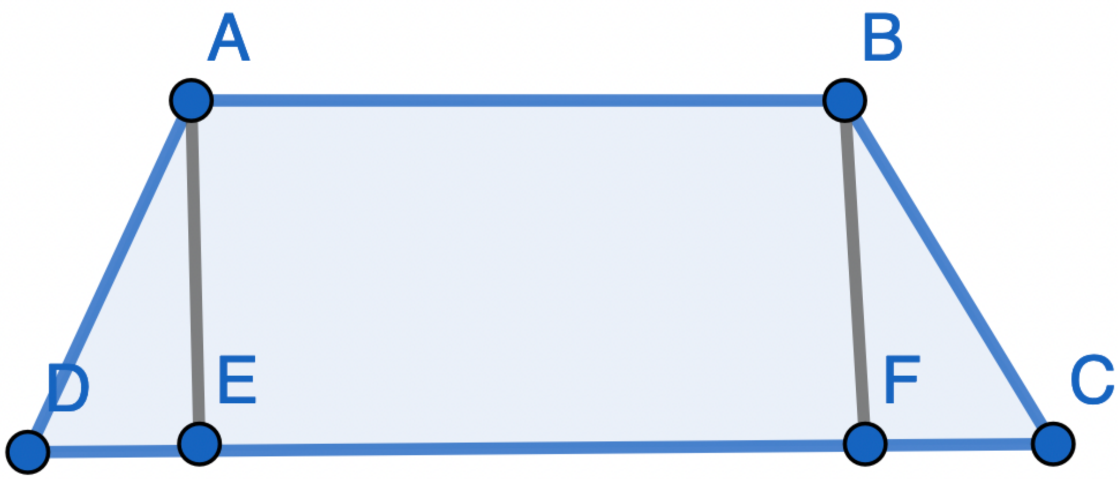
Example 39.



Given the above diagram, where segments DE and CB are perpendicular to the base AB. DE = 4 and BC = 20. If AE = 3, what is the length of DC?

Solution: Because DE and CB are perpendicular to the base AB, then they are parallel. This implies that triangles AED and ABC are similar. Because DE = 4 and BC = 20, The Proportion of the small to large triangle is 1 to 5. By the Pythagorean Theorem, the small triangle is a 3-4-5 right triangle with AD = 5 and therefore AC = 5 * 5 = 25. **Finally DC = AC – AD = 25 – 5 = 20**

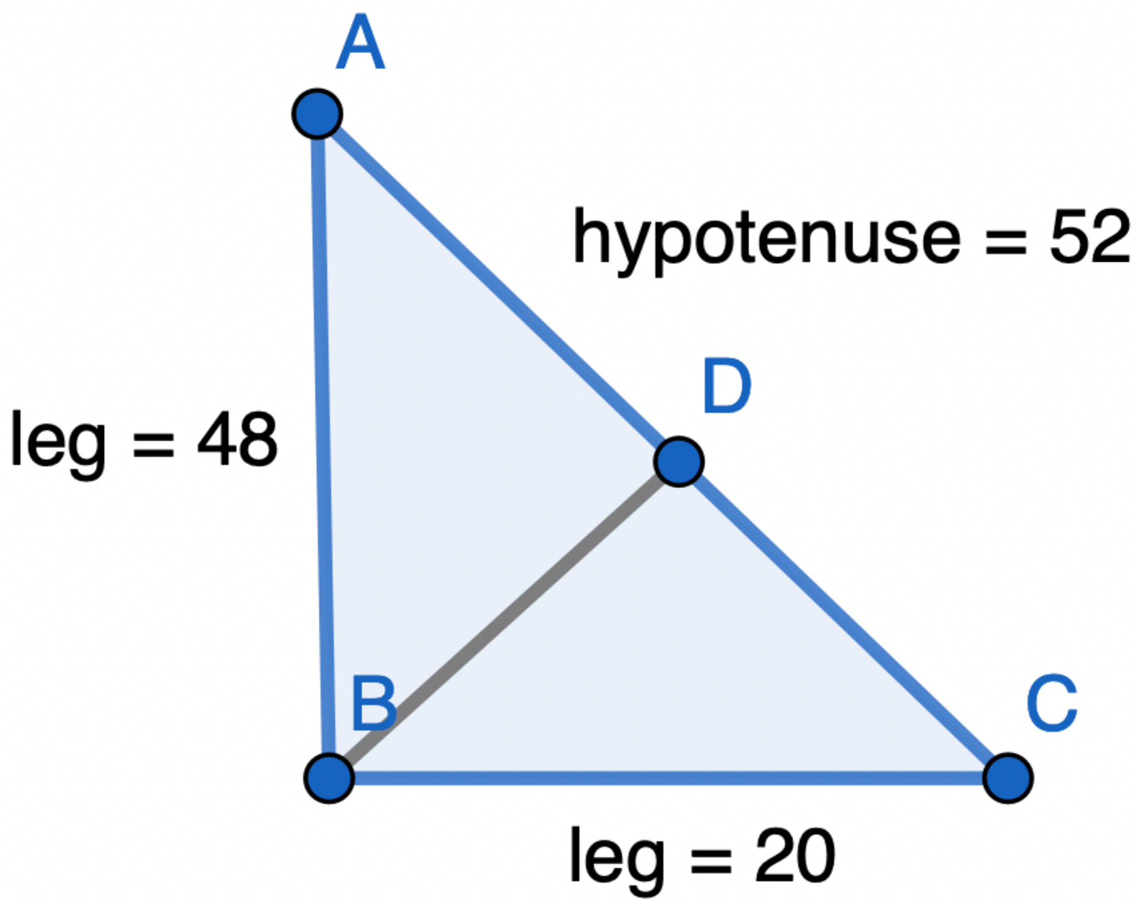
Example 40.



Given trapezoid DABC with AE and BF perpendicular to base DC. IF <ADE = 60°, what is the measure of <DAB?

Solution: Since DEA is a right triangle with <ADE = 60°, then <DAE = 30° and <EAB = 90° (EABF is a rectangle). Therefore, **<DAB = 30° + 90° = 120°**

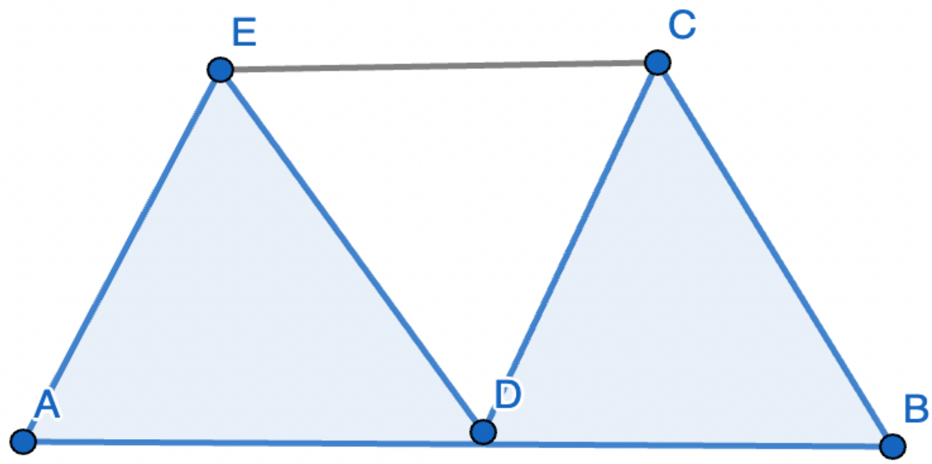
Example 41.



Given the above right triangle ABC with BD perpendicular to AC. What is the length of segment DC?

Solution: These circumstances fit geometrys Altitude on Hypotenuse. If we let segment DC be the variable x, then $20^2 = 52x$ (the rest of the Theorem states that $48^2 = 52 * AD$ and lastly $BD^2 = (52-x)x$. Therefore segment DC = 400/52 = 100/13

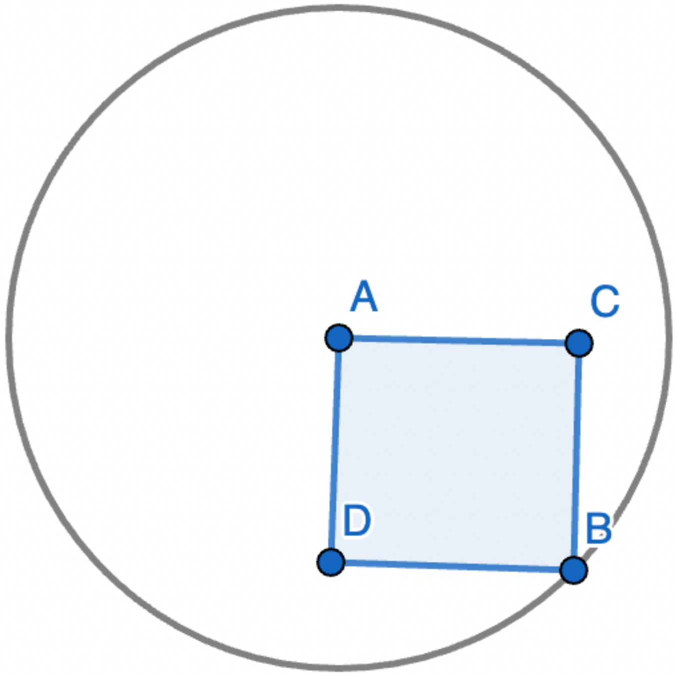
Example 42.



Given: the above trapezoidal figure comprised of 3 congruent equilateral triangles. If the area of the trapezoid = $3\sqrt{3}$ is the perimeter of the trapezoid?

Solution: Each of the three triangles has an area of $\sqrt{3}$. If each side of any one of the triangles is represented by x, then using the formula: area of equilateral triangle = $x^2\sqrt{3}/4$. Therefore: $x^2\sqrt{3}/4 = \sqrt{3}$ and $x^2 = 4$. Therefore the $x = 2$ and since there are 5 equal sides of trapezoid, the perimeter = 5 * 2 = 10.

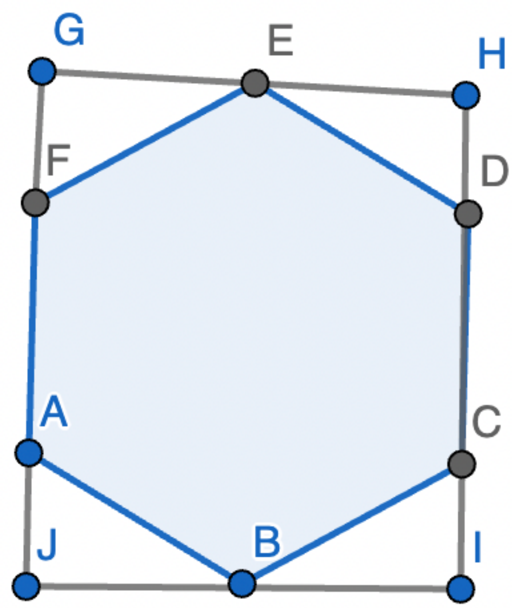
Example 43.



Given a circle with center A and square ACBD with vertex B on the circumference. If the area of the square is 16 what is the area of the circle?

Solution: If the area of the square is 16, then each side is 4. If we consider that ACB form the sides of a right triangle, then this triangle is a 45-45-90 triangle and if a leg = 4 than the hypotenuse (which is also the radius of the circle) equals $4\sqrt{2}$. Hence the area of the circle equals $(4\sqrt{2})^2\pi = 32\pi$.

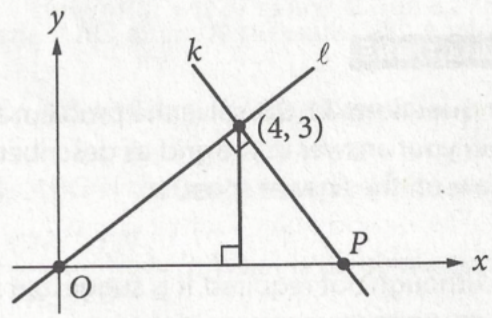
Example 44.



Given the regular pentagon ABCDEF inscribed in the square FGHI. If the GH = 4 units, what is the area of the pentagon?

Solution: using the formula for the sum of the angles of a regular polygon with n sides = 180(n – 2) and divide this by 6 we get the measure of the six angles = 120. By symmetry, this implies that <GEF = 30°. Therefore, triangle FGE is a 3-60-90 triangle. Since leg GH = 2, leg FG = 2/√3. Therefore the area of triangle FGE = ½(2/√3)(2) = 2/√3. Adding the four corner triangles and subtracting from the area of square: 16 – 8/√3 = (16√3 – 8)/√3

Example 45.

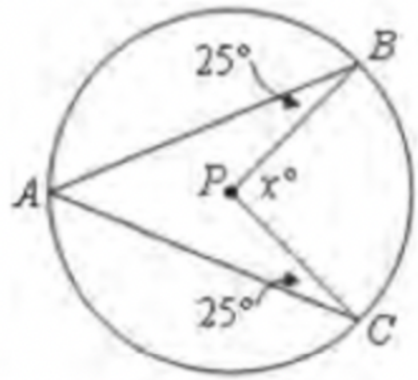


In the xy-plane above, lines k and ℓ are perpendicular. What is the x-coordinate of point P ?

Solution: This is another altitude on hypotenuse problems (see ex.41) The height of the altitude is 3, the y coordinate of (4,3). If we label by Q the point where the altitude intersects the base OP, then $3^2 = OQ * QP$ and since OQ = 4 (the x-coordinate of (4,3)), then 9 = 4QP.

Therefore, QP = 9/4 and P = 4 + 9/4 = 25/4.

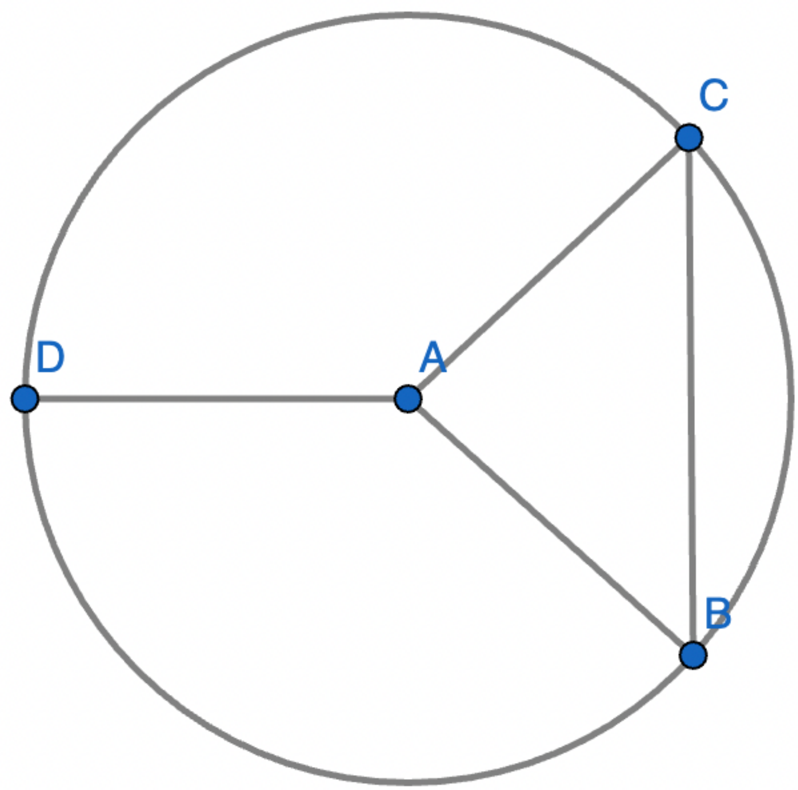
Example 46.



Point *P* is the center of the circle in the figure shown. What is the value of *x* ?

Solution: Draw the radius from P to A. This creates two isosceles triangles ABP and ACP. Both angles <B and <BAP are 25°. Therefore, angle <APB = 180° – 50° = 130°. The same can be said for <APC. Finally, small angle <BPC = 360 – (130 +130) = 100°

Example 47.



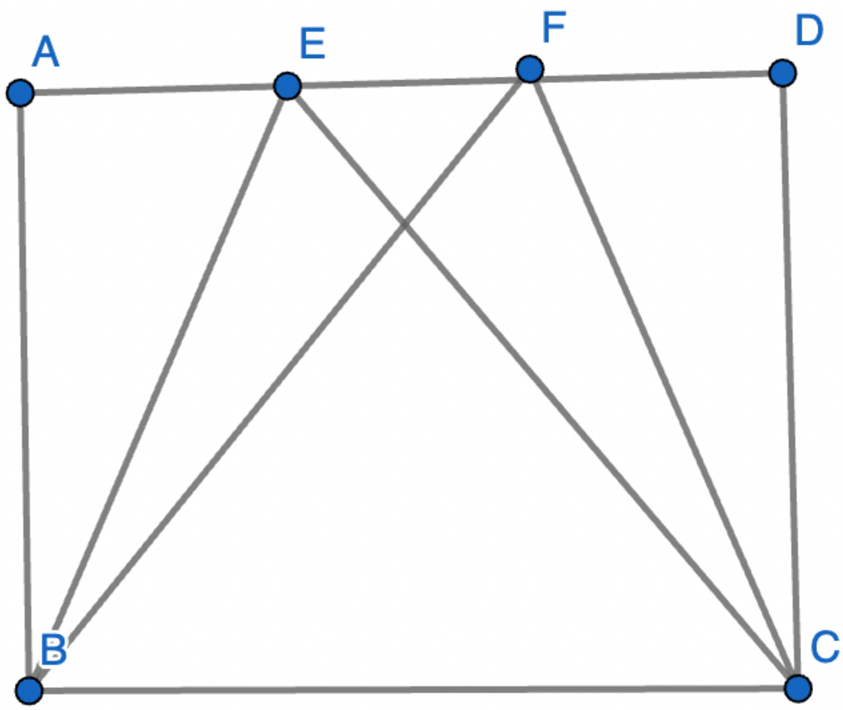
Circle with center A is given above. Angle <CAB equals 80 degrees. If segment DA = 8 units, what is the length of minor arc CB ?

Solution: Determine the circle's circumference = 64Π. The minor arc has degree measure to the central angles (80°). For the proportion with x = the minor arc CB.

x/(64Π) = 80°/360

Therefore, x = (2/9)(64 Π) = 128Π/9

Example 48.



ABCD is a rectangle. Segment AD = 3 units and segment DC = 6 units. Points E and F trisect line segment AD. Segment BF intersects segment CE at point G (not shown). What is combined area of triangles BEG and CFG ?

Solutions: This can be solved by brute force using linear algebra to find the equations of lines EC and FB and then the coordinate of their intersection at point G. Finally we can subtract the triangles EGF and BGC from the Trapezoid BERC to get our desired areas.

But this is overkill. Instead from geometry we see that triangles EBF and BGC are similar and therefore all linear components, specifically the altitudes are in proportion. Therefore, the heights are in the ratio of 1 and 3 respectively and bases are 1 and 4 respectively. Therefore, the areas are: ½BH = ½(1 * 2) = 1 and ½(3 * 4) = 6 respectively. Subtract the sum of the triangles from the trapezoid BERC to get the answer.